

NONRELATIVISTIC MODEL FOR $b\bar{b}$ QUARKONIA *

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Abstract

Experimental data for $b\bar{b}$ quarkonia have been compared with the predictions of a variety of nonrelativistic quark models. It is found that a potential $a\sqrt{r} - b/r + \text{const}$ gives good agreement, while many others do not. Some implications of this observation are discussed.

1 Introduction

According to a recent review[1] the masses of the $b\bar{b}$ quarkonia below twice the mass of the B meson are successfully predicted by a variety of potential models. The average error of the predictions is typically a few MeV — 2.3 MeV in the best case[2]. Some of these models include relativistic corrections, but if we replace the masses of the χ multiplets by their centres of gravity, the relativistic corrections are not crucial for the success. This result is puzzling. From fine splittings, which are a purely relativistic effect, or from the differences between typical kinetic energies evaluated relativistically and nonrelativistically, one finds that the relativistic corrections should be of some tens of MeV. The standard explanation is that what is called the nonrelativistic Hamiltonian, is in fact an effective Hamiltonian with the

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parameters modified by relativistic corrections. This rises the question: is it possible to reproduce the data within errors using such an effective Hamiltonian? Since experimental errors on the masses are of the order of 0.3 MeV, even the "successful" models correspond to very low confidence levels as evaluated from the χ^2 test. Thus in order to answer the question one should do better. Moreover, there are also other data, e.g. leptonic widths and electric dipole transition probabilities, which could and should be fitted. Quark - antiquark potentials derived from studies of quarkonia are used in a variety of applications. Let us mention as examples applications to heavy light systems[3], to $b\bar{c}$ mesons[4], and to $t\bar{t}$ production[5]. Therefore, improving on existing potentials seems useful. On the other hand, as is well known, two body relativistic quantum mechanics does not exist yet. If no nonrelativistic potential can reproduce the data, this may be a valuable hint on how to construct a relativistic theory.

2 The model

There is no standard nonrelativistic quarkonium model. The Schrödinger equation is

$$-\frac{1}{m_b}\vec{\nabla}^2\psi + V(r)\psi = E\psi, \quad (1)$$

but the quark mass m_b and the potential $V(r)$ vary from paper to paper. Fortunately, most proposals for the potential either are of the form

$$V(r) = -ar^{-\alpha} + br^\beta + Ct, \quad (2)$$

where a, b, α, β, Ct are nonnegative constants, or are numerically very well approximated by such formulae. Let us mention as examples the Cornell potential[6] with $\alpha = \beta = 1$, the potential of Lichtenberg and collaborators[7] with $\alpha = \beta = 0.75$, the potential of Song and Liu[8] with $\alpha = \beta = 0.5$, the logarithmic potential of Quigg and Rosner[9] corresponding to $\alpha = \beta \rightarrow 0$ and the potential of Martin[10] corresponding to $\alpha = 0, \beta = 0.1$. Other potentials, which have sometimes quite different analytic forms, become very close to potentials of this type, when their parameters are adjusted to fit the data. We have checked that in particular for the very successful Indiana potential[11] and for the famous Richardson potential[12]. We found that

it is enough to impose on all these potentials the condition that they fit the energy spectrum as well as possible within the freedom described below. This is related to the suggestion of Eichten and Quigg, who concluded from their inverse scattering analysis[13] that the first few $L = 0$ energy levels and the corresponding leptonic widths (not used by us, but we use instead the energies of the $L = 1$ levels) to a large extent determine the potential in the region relevant for the quarkonium calculations. Note, however, that we have no proof that a completely different potential would not fit the data as well.

In order to make our search for a satisfactory potential more effective, we introduced a scaled variable $\rho = \lambda r$. In terms of this variable the Schrödinger equation (1) with potential (2) can be reduced to the form

$$-\vec{\nabla}^2\psi(\rho) + \mathcal{V}(\rho)\psi(\rho) = \mathcal{E}\psi(\rho), \quad (3)$$

where

$$\lambda = \left(\frac{b}{a}\right)^{\frac{1}{\alpha+\beta}}, \quad (4)$$

$$\mathcal{V}(\rho) = C(\rho^\beta - \rho^{-\alpha}), \quad (5)$$

$$\mathcal{E} = \frac{m_b}{\lambda^2}(E - Ct), \quad (6)$$

$$C = m_b a \lambda^{\alpha-2}. \quad (7)$$

Thus, for given values of the parameters α, β the model has four free parameters, which can be chosen as m_b , λ , Ct , and C . Our idea is to choose a set of observables, which depend only on the parameter C . Parameter C is chosen so as to minimize χ^2 for the observables chosen. Then other observables can be used to determine the remaining parameters of the model, but this does not influence the quality of the fit. Using this procedure for the potentials proposed in the literature we got significant improvements of the fits, but none was quite satisfactory.

3 Observables

As observables we have chosen

$$b_1 = \frac{M(2S) - M(1S)}{M(3S) - M(1S)} = 0.6290 \pm 0.0005, \quad (8)$$

$$b_2 = \frac{M(3S) - M(2P)}{M(2S) - M(1P)} = 0.774 \pm 0.006, \quad (9)$$

$$b_3 = \frac{M(2S) - M(1P)}{M(2S) - M(1S)} = 0.219 \pm 0.001, \quad (10)$$

$$b_4 = \frac{|\psi_{2S}(\vec{0})|^2}{|\psi_{1S}(\vec{0})|^2} = 0.492 \pm 0.111, \quad (11)$$

$$b_5 = \frac{|\psi_{3S}(\vec{0})|^2}{|\psi_{1S}(\vec{0})|^2} = 0.433 \pm 0.071, \quad (12)$$

$$b_6 = |\psi_{1S}(\vec{0})|^{\frac{2}{3}} \langle 1P|r|2S \rangle = 2.29 \pm 0.16, \quad (13)$$

$$b_7 = |\psi_{1S}(\vec{0})|^{\frac{2}{3}} \langle 2P|r|3S \rangle = 1.59 \pm 0.15, \quad (14)$$

$$b_8 = \frac{\langle 1S|r|2P \rangle}{\langle 2S|r|2P \rangle} = 0.110 \pm 0.009. \quad (15)$$

The observables b_1, \dots, b_8 depend on the parameter C , but not on the other three parameters. Thus the χ^2 distribution corresponds to seven degrees of freedom. All the numerical values are calculated from the data given in the 1994 Particle Data Group Tables[14]. There are some problems with the observables b_6 and b_7 , however. The formulae connecting the measured leptonic widths to the calculated $|\psi_{nS}(\vec{0})|^2$ contain a radiative correction factor. This is $1 - \frac{16\alpha_s}{3\pi}$ and we put it equal 0.7. Since neither the exact value of α_s , nor the effect of the higher order terms is known, this number is uncertain by perhaps some 20%.. This uncertainty very probably cancels in b_4 , b_5 and b_8 , but for b_6 and b_7 it introduces a common factor, which deviates from unity by some seven per cent. This uncertainty has not been included in the errors. Note also that the simple dipole formulae are much less reliable for quarkonia than for atoms.

Once the parameter C is chosen so as to reproduce as well as possible the observables b_1, \dots, b_8 , the parameters λ , m_b and Ct can be calculated from the experimental value the leptonic width of the $\Upsilon(1S)$ state, which yields $|\psi_{1S}(\vec{0})|^2$, from the mass difference $M(3S) - M(1S)$ and from the mass $M(1S)$. This calculation has no bearing on the quality of the model,

except that perhaps values of m_b too far from 5 GeV would make the model unpalatable.

4 Results and conclusions

After some exploration in the α, β plane, we have found a positive answer to our question. For $\alpha = 1$, $\beta = 0.5$ we find for $C = 0.9315$ the value $\chi^2 = 6.5$, which is very satisfactory for the seven degrees of freedom. This solution corresponds to the potential in eq. (1) given by

$$V(r) = 0.70585 \left(\sqrt{r} - \frac{0.46122}{r} \right), \quad (16)$$

where $V(r)$ and r are in GeV. The corresponding quark mass $m_b = 4.79333$ GeV is quite reasonable. For $r \rightarrow 0$ our potential has the r^{-1} dependence corresponding to one gluon exchange. With present data, however, we have no evidence for the additional factor $1/\log(\Lambda r)$, which according to QCD should be introduced by the running of the coupling constant. The expected part of the potential linear in r is not seen. Probably the bottomonia are too small to reach sufficiently far into the asymptotic region of linear confinement. Perhaps a more flexible potential would exhibit the linear part.

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